

On discrete symmetries and relic radiation anisotropy

M.V.Altaisky

*Space Research Institute RAS, Profsoyuznaya 84/32, Moscow, 117997, Russia; and Joint
Institute for Nuclear Research, Joliot-Curie 6, Dubna, 141980, Russia*

altaisky@mx.iki.rssi.ru

and

N.E.Kaputkina

*National University of Science and Technology "MISiS", Leninsky prospect 4, Moscow,
119049, Russia*

nataly@misis.ru

ABSTRACT

It is argued that large scale angle correlations of the Cosmic Microwave Background Radiation (CMBR) temperature fluctuations measured by Wilkinson Microwave Anisotropy Probe (WMAP) mission may have a trace of discrete symmetries of quantum gravity

Subject headings: cosmic background radiation, large-scale structure of universe

The standard cosmological model, the Λ CDM, assumes the matter to be forever expanding in utmost flat universe under the pressure of dark energy. In early Universe, before the recombination of electrons and protons and the formation of galaxies, the electron-proton plasma was assumed homogeneous. The formation of galaxies was considered as a consequence of gravitational instability of small initial density fluctuations $\delta\rho/\rho \sim 10^{-5}$, constrained by the results of WMAP mission (Hinshaw et al. 2009). In post-recombination epoch the gas of neutral atoms became utmost transparent to photons. The CMBR, with the present temperature 2.7K, was no longer scattered by the matter (except for the scattering on the free electrons of galaxies (Sunyaev and Zeldovich 1970)), and therefore carries invaluable information on the early Universe.

The Λ CDM model is an inflationary model, which depends on six parameters: the barion density $\Omega_b h^2$, the cold dark matter density $\Omega_c h^2$, a cosmological constant Ω_Λ , the spectral

index of fluctuations n_s , the scalar fluctuation amplitude $\Delta_{\mathcal{R}}^2$, and the optical reionization depth τ (J. Dunkley and Wright 2009; D. Larson and Wright 2011). The model assumes a nearly Gaussian spectrum of initial fluctuations with statistical isotropy over the sky (Partridge and Wilkinson 1967). Any deviation from isotropy would be a challenge for a new physics (Efstathiou 2003; Tegmark et al. 2003).

Although up to the recent WMAP data the Λ CDM model remains a good fit of the observed microwave sky (E. Komatsu and Wright 2011), the key feature displayed by WMAP (C. L. Bennett and Wright 2011) and previous experiments (Bennett et al. 1996; Smoot et al. 1992; Strukov et al. 1993), is that the Universe is isotropic in the mean, but anisotropic in correlations. This means there are no preferable directions, but there are preferable angles of correlations. The observation of such anisotropy is contr-intuitive from classical physics point of view, but seems quite natural in quantum mechanics, like that of the Einstein-Podolsky-Rosen (EPR) correlations (Einstein et al. 1935). The anisotropy in angle correlations has been receiving constant attention since discovered (Smoot et al. 1992). WMAP mission itself was designed to use the observed correlations of fluctuations to put narrower constraints on cosmological parameters after those obtained by the previous COsmic Background Explore (COBE) mission (Bennett et al. 1996, 2003).

Two main types of data are available to constrain the cosmological parameters: (i) the observed correlations in the distribution of galaxies (Peebles and Groth 1975); and (ii) the observed correlations of the relic radiation (C. L. Bennett and Wright 2011). The main instrument for the analysis of both data type remains the decomposition with respect to the representations of the $SO(3)$ group of rotations in \mathbb{R}^3 . This means the n -point correlation function $\langle u^{\beta_1} \dots u^{\beta_n} \rangle$ transforms under the spacial rotations according to the law

$$\langle u'^{\alpha_1} \dots u'^{\alpha_n} \rangle = \Lambda_{\beta_1}^{\alpha_1} \dots \Lambda_{\beta_n}^{\alpha_n} \langle u^{\beta_1} \dots u^{\beta_n} \rangle \quad (1)$$

where Λ is the matrix of $SO(3)$ rotation in appropriate representation. Thus a scalar remains invariant under rotations; the n -point correlation function of a vector field thus transforms according to the direct product $\underbrace{\Lambda \otimes \dots \otimes \Lambda}_{n \text{ times}}$, etc.

In the COBE and the WMAP data the full sky map was decomposed into a series of spherical harmonics $Y_{lm}(\mathbf{n})$, which form irreducible representations of $SO(3)$ group in \mathbb{R}^3 :

$$T(\mathbf{n}) = \sum_{l=0}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\mathbf{n}), \quad \text{with } a_{lm} = \int d\mathbf{n} T(\mathbf{n}) \overline{Y_{lm}(\mathbf{n})}, \quad (2)$$

where \mathbf{n} is the unit direction vector. If the CMBR anisotropy is driven by a Gaussian random process with random phases and zero mean, then

$$\langle a_{lm} \overline{a_{l'm'}} \rangle = C_l \delta_{ll'} \delta_{mm'}, \quad (3)$$

where C_l is the angular power spectrum. The statistical distribution of the a_{lm} coefficients has been thoroughly studied by the WMAP team, with no significant deviation of the gaussianity found (E. Komatsu and Wright 2011).

At large angles (low multipoles) the main attention has been paid to the quadrupole component, which was studied in the framework of different models since first measured by COBE (Bennett et al. 1996; Hinshaw et al. 1996; Efstathiou 2003; de Oliveira-Costa and Tegmark 2006; C. L. Bennett and Wright 2011).

The pair correlator of two events separated by the angle α

$$C(\alpha) = \langle \Delta T(\theta) \Delta T(\theta + \alpha) \rangle \quad (4)$$

is related to the subgroup $SO(2)$ of the rotation group $SO(3)$. The $C(\alpha)$ averages over the orientation of the observation plane. Since first measured by COBE the pair correlator (4) has been parametrized in the form (Wright 1992)

$$C(\alpha) = A + B \cos \alpha + C_M^0 \exp \left[-\frac{\alpha^2}{2\sigma^2} \right],$$

although the locations of autocorrelation function maxima at 0 and $\frac{2\pi}{3}$ and the minima at $\frac{\pi}{3}$ and π suggest another parametrization (Altaisky et al. 1996):

$$C(\alpha) = A + B \cos 3\alpha + C_M^0 \exp \left[-\frac{\alpha^2}{2\sigma^2} \right], \quad (5)$$

where A, B, C_M^0, σ are constants.

If the correlation function $C(\alpha)$ has a maximum around $\alpha = \frac{2\pi}{3}$ and the minimum around $\alpha = \frac{\pi}{3}$, see Fig.5 of (C. L. Bennett and Wright 2011), the obvious subgroup of $SO(2)$, which can explain such behavior, is the group C_3 of the rotations $R_k = R_1^k = R \left(\frac{2\pi k}{3} \right), k = 0, 1, 2$:

$$C_3 = \{ E, R_1, R_1^2 \}. \quad (6)$$

The simplest explanation of the presence of C_3 group in the CMBR temperature fluctuations was given in (Altaisky et al. 1996). This explanation is in considering simplicial quantum gravity (Ambjørn and Jurkiewicz 1992; Agishtein and Migdal 1992), which enables to describe pure gravity in a consistent way. The solution with coupling to matter fields was found for two-dimensional case (Brézin and Kazakov 1990; Boulatov et al. 1986). Starting from Boulatov-Ooguri lattice gravity models (Boulatov 1992; Ooguri 1992), which substitute the integral over all possible geometries by a discrete state sum over all possible triangulations $Z = \sum_T C_T e^{-S(T)}$, where the action $S(T)$ counts the number of simplexes, the theory has merged into spin-foam quantum gravity (Baret and Crane 1998). The spin foam

has a natural interpretation of quantized space-time based on discrete symmetry groups (Perez and Rovelli 2001; Crane et al. 2001).

The assumption of the gaussianity of initial density fluctuations, being purely classical, is not self-sufficient. It leaves the question where from the smooth manifold of Friedmann-Robertson-Walker universe has originated. The formation of a smooth manifold by no means follows the Big Bang hypothesis. Instead, the formation of a smooth manifold should arise from quantum gravity models. The dynamical triangulation in these models allows for a smooth manifold in the limit of the infinite number of simplexes, preserving the initial discrete symmetries at a quantum level.

A simple toy-model, which assumes the dynamical formation of fractal space with a symmetry group of simplex, which covers a d -dimensional sphere, has been proposed in (Altaisky et al. 1996). The model assumes the dynamical creation of discrete geometry starting from a "point" Universe. An alternative model is based on the discrete symmetry group of curved dodecahedral Poincare space. In a curved Poincare dodecahedral space the spherical dodecahedrons, used as building cells, have the edge angles exactly $\frac{2\pi}{3}$, which may explain the correlation maximum observed by WMAP mission (Luminet et al. 2003).

If we base on symmetry only, without any special assumptions on geometry, we ought to construct n -point correlation functions according to the general theory of group representations, see e.g. (Hamermesh 1989). So the Λ_β^α matrices in (1) must be the matrices of a given representation of symmetry group. In the simplicial fractal model of Altaisky et al. (1996) the considered symmetry is the abelian group C_3 . The C_3 group (6) has three non-equivalent irreducible representations $T^{(1)}$, $T^{(2)}$, $T^{(3)}$, the characters of which are shown in Table 1 in Appendix.

The representation $T^{(1)}$ is singlet. It describes the functions not affected by C_3 transformations. Two other representations $T^{(2)}$ and $T^{(3)}$ correspond to the functions subjected to the left and the right phase rotations under the C_3 transformations. If the functions u_i, u_j, u_k, \dots transform according to representations $T^{(i)}$, $T^{(j)}$, $T^{(k)} \dots$, respectively, the n -point correlation function transforms according to the direct product $T^{(i)} \otimes T^{(j)} \otimes T^{(k)} \otimes \dots$

The decomposition of the direct product into a sum of irreducible representations is casted in the form

$$T^{(\alpha)} \otimes T^{(\beta)} = \oplus_\gamma m_\gamma T^{(\gamma)} \quad (7)$$

with the weights m_γ is determined by standard formula (Hamermesh 1989)

$$m_\gamma = \frac{1}{g} \sum_{p \in G} \overline{\chi_p^\gamma} \chi_p^\alpha \chi_p^\beta, \quad (8)$$

where g is the number of elements in the group G , χ_p^α is the character of the element p in the representation $T^{(\alpha)}$. For the group C_3 the decomposition (7) has a simply-reducible form

$$\begin{aligned} T^{(1)} \otimes T^{(1)} &= T^{(1)}, & T^{(1)} \otimes T^{(2)} &= T^{(2)}, & T^{(1)} \otimes T^{(3)} &= T^{(3)}, \\ T^{(2)} \otimes T^{(2)} &= T^{(3)}, & T^{(3)} \otimes T^{(3)} &= T^{(2)}, & T^{(3)} \otimes T^{(2)} &= T^{(1)}. \end{aligned} \quad (9)$$

For definiteness, let us consider the field u , which transforms according to the $T^{(2)}$ representation, then

$$\langle u \rangle \sim T^{(2)}, \quad \langle uu \rangle \sim T^{(3)}, \quad \langle uuu \rangle \sim T^{(1)}. \quad (10)$$

As it follows from the (10), starting from the field with nontrivial transformation properties, either $T^{(2)}$ or $T^{(3)}$, the singlet states, i.e., those invariant under any C_3 transformation, can be obtained only for triple correlator $\langle uuu \rangle$, rather than for pair correlator $\langle uu \rangle$.

The singletness of the triple correlations in (10) is an interesting fact for the C_n group is the a point subgroup of the $SO(3)$. The other point subgroups of $SO(3)$ related to polyhedra are nonabelian groups. In the simplicial model of (Altaisky et al. 1996) the next to C_3 hierarchic level has the symmetry group of tetrahedron T_d , which includes C_3 as a subgroup. T_d comprises 3 axis of second order (C_2) and also (C_3) rotations R_1 and R_1^2 for each of four faces. T_d has 4 irreducible representations (3 one-dimensional representations: T^1, T^2, T^3 , and one three-dimensional, T^4 , which stand for rotations). Using the character table of T_d group (Table 2 in Appendix), we can see that either of representations T^1, T^2, T^3 yield a singlet in triple correlations, while T^4 does not. The application of (8) gives the following decomposition table for T_d :

$$\begin{aligned} T^{(1)} \otimes T^{(\alpha)} &= T^{(\alpha)}, & T^{(4)} \otimes T^{(\beta)} &= T^{(4)}, & \alpha &= 1, 2, 3, 4; \beta = 1, 2, 3, \\ T^{(2)} \otimes T^{(2)} &= T^{(3)}, & T^{(3)} \otimes T^{(2)} &= T^{(1)}, & T^{(3)} \otimes T^{(3)} &= T^{(2)}, \\ T^{(4)} \otimes T^{(4)} &= T^{(1)} + T^{(2)} + T^{(3)} + 2T^{(4)}. \end{aligned} \quad (11)$$

As it can be seen from multiplication table (11), if we take a field u in one-dimensional representation $T^{(2)}$ the relations (10) hold for tetrahedron group. This confirms the conclusions of (Altaisky et al. 1996) that the observed C_3 extrema in pair correlations may be manifestations of underline T_d symmetry.

To test the presence of discrete symmetries, either C_3 or T_d , we can study the behavior of the three-point correlation functions, specially that with equal angles between the legs $\alpha = \frac{2\pi}{3}$. The deviations from singlet, possibly due to polarization, may be important to test the discrete symmetry of the Universe at quantum gravity stage.

The suggested discrete symmetry may in some sense be similar to that of quantum chemistry of NH_3 and CH_4 molecules (C_3 and T_d symmetry, respectively). A molecule "as

it is” before any measurement have no preferable direction, but then measured or participate in chemical reaction, the discrete symmetry is displayed.

To stress the significance of C_3 symmetry we fit the recently released 7 year WMAP data (Fig.5 of C. L. Bennett and Wright (2011)) by the equation (5). The result is shown in Fig. 1. Clearly in the region $\frac{2\pi}{3} \leq \alpha \leq \pi$ the decreasing behavior of the equation (5) better

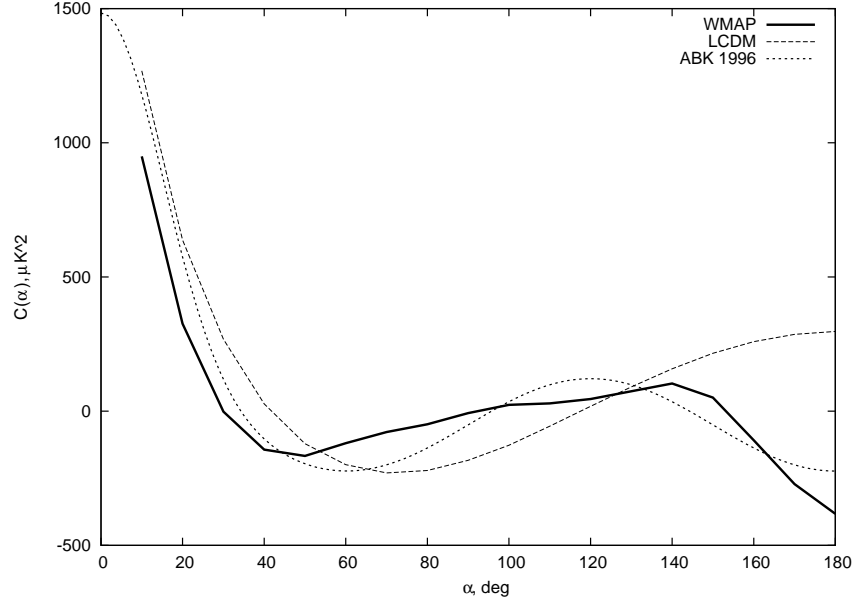


Fig. 1.— Fit of the C_3 symmetry assumption (Eq.8 of Altaisky et al. (1996)) onto WMAP data. The solid line indicates WMAP temperature fluctuations correlation function ; the dashed line shows Λ CDM prediction – both lines from Fig.5 of C. L. Bennett and Wright (2011). By dotted line we indicated the prediction of the equation (5)

fits WMAP correlation function then the Λ CDM curve.

The next desired step in study of the discrete symmetries of CMBR induced by underline quantum gravity structure would be the measurement of triple correlations $\langle uuu \rangle$ and comparison of the resulting function with triple correlations of galaxies density. The observed pattern of galaxy clustering satisfies the hierarchic form (Fry 1984):

$$\zeta_{x_1 x_2 x_3} = Q(\xi_{x_1 x_2} \xi_{x_1 x_3} + \xi_{x_1 x_2} \xi_{x_2 x_3} + \xi_{x_1 x_3} \xi_{x_2 x_3}),$$

where $\xi_{x_i x_j}$ is the pair correlator of the observed galaxy density, with statistical estimations of the Q values being in the limits $0.85 \div 1.24$ (Peebles and Groth 1975; Groth and Peebles 1977).

To conclude with we would like to emphasize that the manifestation of quantum gravity symmetry in triple correlations is in some sense inverse to the measuring EPR correlations

(Einstein et al. 1935). In EPR the spin correlations are measured by *outside* observer, in the CMBR studies we measure correlations from *inside* the Universe. In both cases there are no *a priori* arguments for ergodicity, and in both cases the measurements should be performed *in situ*. The offline measurements by separate counters may just return the mean number of photons from each direction.

Acknowledgements

The authors are thankful to Drs. G. Hinshaw and D.P.Skulachev for useful discussions. The research was supported in part by the RFBR Project 11-02-00604-a and the Program of Creation and Development of the National University of Science and Technology "MISiS".

REFERENCES

- Agishtein, M. and Migdal, A. (1992). *Modern Phys. Lett. A*, 7(12):1039–1061.
- Altaisky, M., Bednyakov, V., and Kovalenko, S. (1996). *Int. J. Theor. Phys.*, 35(2):253–261.
- Ambjørn, J. and Jurkiewicz, J. (1992). *Phys. Lett. B*, 278(1-2):42–50.
- Barett, J. and Crane, L. (1998). *J. Math. Phys.*, 39:3296–3302.
- Bennett, C., et al. (1996). *ApJ*, 464:L1–L4.
- Bennett, C. L., et al. (2003). *ApJ*, 583(1):1.
- Boulatov, D. (1992). *Modern Phys. Lett. A*, A7(18):1629–1646.
- Boulatov, D., et al. (1986). *Nuclear Physics B*, 275(4):641–686.
- Brézin, E. and Kazakov, V. A. (1990). *Phys. Lett. B*, 236(2):144–150.
- Bennett, C.L., et al. (2011). *ApJS*, 192(2):17.
- Crane, L., Perez, A., and Rovelli, C. (2001). *Phys. Rev. Lett.*, 87(18):181301.
- Larson, D., et al. (2011). *ApJS*, 192:16.
- de Oliveira-Costa, A. and Tegmark, M. (2006). *Phys. Rev. D*, 74(2):023005.
- Komatsu, E., et al. (2011). *ApJS*, 192:18.

- Efstathiou, G. (2003). MNRAS, 343:L95–L98.
- Einstein, A., Podolsky, B., and Rosen, N. (1935). *Phys. Rev.*, 47:777–780.
- Fry, J. (1984). ApJ, 279(2):499–510.
- Groth, E. and Peebles, P. (1977). ApJ, 217:385.
- Hamermesh, M. (1989). *Group theory and its application to physical problems*. Dover Publications.
- Hinshaw, G., et al. (1996). ApJ, 464:L17.
- Hinshaw, G., et al. (2009). ApJS, 180:225–245.
- Dunkley, J., et al. (2009). ApJS, 180:306.
- Luminet, J.-P., et al. (2003). Nature, 425(6958):593–595.
- Ooguri, H. (1992). *Modern Phys. Lett. A*, A7(30):2799–2810.
- Partridge, R. and Wilkinson, D. (1967). Phys. Rev. Lett., 18:557–559.
- Peebles, P. and Groth, E. (1975). ApJ, 196:1–11.
- Perez, A. and Rovelli, C. (2001). Phys. Rev. D, 63(4):041501.
- Smoot, G. and et al. (1992). ApJ, 396:L1.
- Strukov, I., et al. (1993). *Phys. Lett. B*, 315(1-2):198–202.
- Sunyaev, R. A. and Zeldovich, Y. B. (1970). Ap&SS, 7(1):3–19.
- Tegmark, M., et al. (2003). Phys. Rev. D, 68(12):123523.
- Wright, E. (1992). ApJ, 396:L13.

Appendix

Characters of irreducible representations of C_3 and T_d groups

	E	R_1	R_1^2
$\chi^{(1)}$	1	1	1
$\chi^{(2)}$	1	$e^{\frac{2\pi i}{3}}$	$e^{\frac{4\pi i}{3}}$
$\chi^{(3)}$	1	$e^{\frac{4\pi i}{3}}$	$e^{\frac{2\pi i}{3}}$

Table 1: Characters of irreducible representations of the C_3 group

	E	$C_2(3)$	$R_1(4)$	$R_1^2(4)$
$\chi^{(1)}$	1	1	1	1
$\chi^{(2)}$	1	1	$e^{\frac{2\pi i}{3}}$	$e^{\frac{4\pi i}{3}}$
$\chi^{(3)}$	1	1	$e^{\frac{4\pi i}{3}}$	$e^{\frac{2\pi i}{3}}$
$\chi^{(4)}$	3	-1	0	0

Table 2: Characters of irreducible representations of the T_d group. The indices in parentheses denote the numbers of elements in each class